## Examples of Mapping "The Foot of the Continental Slope" With "The Surface of Directed Gradient" Algorithm Using NOAA's ETOPO5 Data Base

By

John Overton Bennett President and CEO Paradigm Imaging Inc. 11408 Orchard Green Court Reston, Virginia 20190 703-709-9569 pijob@home.com

### ABSTRACT

I first presented The Surface of Directed Gradient Algorithm, developed by Professor Carl de Boor, at the 1996 GALOS Conference in Bali in a paper entitled "Mapping the Foot of the Continental Slope (FCS) with Spline Smoothed Data Using the Second Derivative in the Gradient Direction (SDG)." Since that time I have run the algorithm on several areas using the ETOPO5 data developed by NOAA. Although there are several bathymetric data bases that have been developed since that of ETOPO5 with a finer grids and more accurate z, water depth values, ETOPO5 is a complete world wide data base on a rectangular grid and thus provides a standard for comparing results for areas in different parts of the world. Its latitude and longitude rectangular grid is the widest at the equator where it is 5 nautical miles. This is a fine enough grid to compute a reasonable first pass at the FCS anywhere in the world. The grid is finer the further the area is from the equator. Most of the ETOPO5 database was obtained from the declassification of some digital United States Navy data sets.

The SDG Algorithm as presented in the paper I gave at the Bali GALOS Conference Bali is a faithful translation of the United Nation's Law of the Sea Article 76 definition of the FCS as "the point of maximum change of the gradient at its base" into a mathematical algorithm. The SDG Algorithm relies on having available an explicit mathematical formula for the sea depth, z, in the area of interest. For such data on a rectangular grid, I have obtained that mathematical formula as a bi-cubic smoothing spline representation to the raw data. On occasion the ETOPO5 data are a bit noisy in some areas and requires some smoothing in order to map the location of the FCS in a meaningful way.

This is not necessarily the method of computing the Foot of the Continental Slope to be used by the United States Government. The examples presented in this paper represent no opinion or position of the United States government. They are merely the observations of the author.

## TEXT

I first presented The Surface of Directed Gradient (SDG) Algorithm, developed by Professor Carl de Boor, at the 1996 GALOS Conference in Bali in a paper entitled "Mapping the Foot of the Continental Slope (FCS) with Spline Smoothed Data Using the Second Derivative in the Gradient Direction (SDG)." (Reference 1,Bennett) Since that time I have run the algorithm on several areas using the ETOPO5 data developed by NOAA. The 1996 GALOS paper was 23 pages in length. I do not feel it gave the proper perspective of the importance of the data smoothing aspect of the NOAA ETOPO5 data set nor emphased the fact that the SDG Algorithm as presented in the paper given at the Bali GALOS Conference Bali is a faithful translation of the United Nation's Law of the Sea Article 76 definition of the FCS as "the point of maximum change of the gradient at its base" into a mathematical algorithm. For this reason I do not feel that the FSC Algorithm can be improved in a digital implementation of the Legal Definition of the FCS although there have several methods presented to compute the FCS. As long as this remains the UN's legal definition the FCS than SDG is as good a method of determining it as is possible.

The purpose of this paper is to present an example to explain how the SDG algorithm computes the FCS without the mathematical equations and theoretical detail in order to make the SDG algorithm more easily understood.

Similarly for the NOAA ETOPO5 data base, although there have several data bases that have been developed since that of ETOPO5 with a finer grids and more accurate z, water depth values, it is a complete world wide data base on a rectangular grid and thus provides a standard for comparing results for areas in different parts of the world. Its latitude and longitude rectangular grid is the widest at the equator where it is 5 nautical miles. This is a fine enough grid to compute a reasonable first pass at the FCS anywhere in the world. A large part of the ETOPO5 database was obtained from the declassification of some United States Navy data sets.

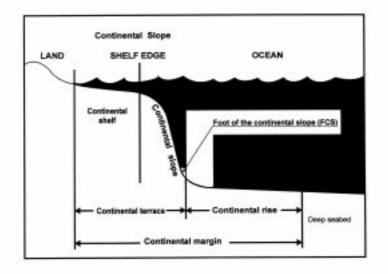
An explicit mathematical bi-cubic spline function is obtained to represent the ETOPO5 bathymetric sea depth, z, in the area of interest. On occasion the ETOPO5 data are a bit noisy in some areas and requires some smoothing of the spline function in order for the SDG to map the location of the FCS in a meaningful way.

Also there are areas of the world where the continental shelf does not readily lend itself to the use of the UN's legal definition of the FCS. In these cases a spline smoothing of the data function can take out the worst of the divergence from the UN model where the legal definition applies to give a spline smoothed approximation to the FCS.

As new technology allows for deeper offshore drilling for hydrocarbons and minerals, coastal countries have more reason to want to extend their mineral rights past the Exclusive Economic Zone (EEZ); thus the location of the FCS will become more important and crucial. The United Nation's Law of the Sea (U.N. LOS) gives the legal definition of the FCS to be, "In absence of evidence to the contrary, the foot of the continental slope shall be determined as the point of maximum change in the gradient" (See Reference 7 pg. 27, Article 27, 4, b.) See figure 1.







Briefly a country's mineral rights may be determined by one of 5 options. (See de Wet, reference 5.)

- 1. 350 nautical miles from the coast line,
- 2. -2500 meter isobath plus 100 nautical miles,
- 3. FCS seaward of the EEZ plus 60 nautical miles,
- 4. 1% rule: points each of which the thickness of the sedimentary rocks is at least 1% of the shortest distance from such point to the FCS,
- 5. 200-nautical mileExclusive Economic Zone (EEZ).
- Let M1=Maximum {1,2}
- Let M2=Maximum {3,4}

then the outer limit of the continental shelf that can be claimed under UN Article 76 is given by:

Maximum [ Minimum { M1, M2 }, 5 ].

For the full legal description of the determination of a country's mineral rights see page 27 of the United Nations publication, reference 7.

Because two (3 and 4) of the four options above are dependent on the location of the FCS, it is clear that it is critical that the FCS be mapped accurately.

The FCS can extend a nation's mineral rights past the 200-nautical mile EEZ where the FCS is seaward of the EEZ of that country's coast, option 3. When this occurs, 60 nautical miles are added seaward of the location of the FCS to determine the extent of the coastal country's mineral rights. It is also possible to even go beyond the 60-nautical mile extension of the FCS where reliable seismic data with good velocity control, gravity, or magnetic geophysical data document a substantial increase in sediment thickness via the 1% rule, option 4. Options 3 and 4 do not have any explicit constraints associated with them; however, options 1 and 2 are constraints that do bound the seaward extension of the mineral rights.

Having assembled documented evidence to make a claim via one or both of the above options 3 or 4, a country would then petition the United Nation's "Commission on the Limits of the Continental Shelf" (CLCS) to extend its continental shelf based upon the maps documenting the evidence to support options 3 or 4 above. The United Nations has provided a useful publication with scientific and technical guidelines for countries submitting claims to the CLCS. (See United Nation's Publication Scientific and Technical Guidelines the Limits of the Continental Shelf: reference 9.) This publication cites the SDG algorithm as a method to compute the FCS.

In any case, the extension of a country's mineral rights can never exceed 350 nautical miles offshore or 100 nautical miles seaward of the -2,500 meter isobath, whichever is the farther seaward. Any mineral rights seaward of a country's mineral's boundary once determined are retained by the United Nations to finance development of third world countries. All coastal countries are entitled to the mineral rights afforded them by the 200-nautical mile EEZ, option 5 above with no need to appeal to options 3 or 4 above.

# **EXAMPLE 1: US ATLANTIC COAST**

An example from the US Atlantic Ocean to illustrate the implementation of option 3 to compute the FCS above will now be given.

What is needed is a worldwide digital bathymetric data set. It is fortunate that the worldwide ETOPO5 data set at its 6-nautical mile grid is about right to allow a first cut at determining any country's FCS. Proper data gathering with proper grid size best suited to accurately characterize a particular country's FCS is now considered the most important aspect of locating their FCS.

NOAA's ETOPO5 data set leaves a bit to be desired in resolution and accuracy; however, because of the high cost involved in obtaining new data, this is a good first cut for a country to decide how much more digital bathymetric data are needed to prepare maps to present to the United Nation's CLCS.

The U.N.'s legal definition of the FCS, "shall be determined as the point of maximum change in the gradient at its base" is very precise; but, it is not easy to

implement on paper bathymetric contour maps. See figure 1 for a cross-section showing the location of the FCS. The legal definition of the FCS almost requires a digital environment for a meaningful implementation.

This definition of the FCS applies also to well-defined continental slopes such as the U.S. Atlantic coast, which has a smooth steep drop off. It does not do well on sub-sea rolling hills type continental slopes such as the North Slope of Alaska west of Prudhoe Bay. For a continental slope with volcanic activity causing much angularity, such as that of Japan, this definition does not apply well at all. For areas such as these, an alternate legal definition is needed.

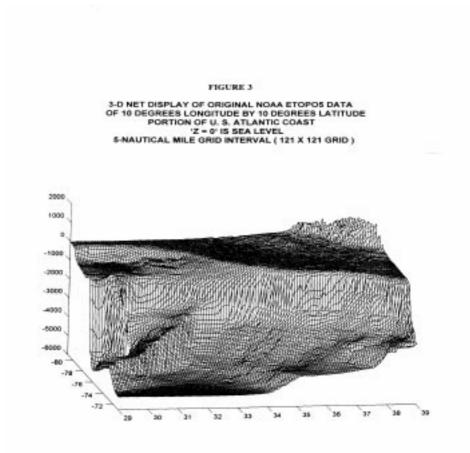
This legal definition was translated to a mathematical algorithm called the Surface of Directed Gradient (SDG) by de Boor and presented by Bennett in references 1 and 2. The input is digital bathymetric data sets on a rectangular grid and the resulting second derivative surface is called the Surface of Directed Gradient (SDG). The crest of the highest ridge of this surface locates the FCS wherever the U.N. legal definition is appropriate.

Other digital methods are also used to compute a second derivative surface from digital bathymetric data using the surface of maximum curvature. See Vanicek references 10 and 11. I gave a paper comparing the two methods at the Advisory Board of the Law of the Sea (ABLOS) meeting September of 1999 in Monaco. The SMC had spurious lobes that got worse the more complicated and the tighter the folds in the test function became. The SMC also required scaling all three of the x, y, and z-axes to the same units to determine the FCS. Once the FSC was determined the SMC required the rescaling of the function back the initial units on the x, y, and z axes for proper mapping of the FCS for the function used. The SDG algorithm had no spurious lobes and requires no scaling to locate the FCS. See reference 3.



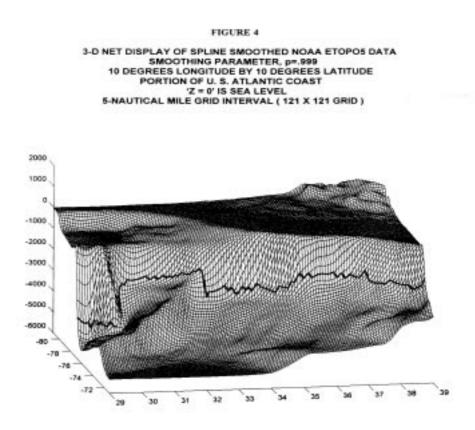
FIGURE 2

An area where the United Nation's legal definition of the FCS applies is illustrated by the U. S. Atlantic Coast in latitude North from Florida at 29 degrees to Delaware at 39 degrees and in longitude West from -81 degrees to -71 degrees. See **figure 2** for a map showing the area of consideration.



**Figure 3** is a 3-D net display of the raw ETOPO5 digital bathymetric data. The smoothing parameter, p, is between zero and including 1. If p=1.0 there is no smoothing. If p is close to zero there is maximum smoothing. In order not to loose the information content of the data set 'p' is bounded such that  $\{.9 . If 'p <.9', there is usually a severe loss of information content. There is no smoothing (p=1) of these raw data in figure 3. For each different area where the FCS is to be determined, different amounts of smoothing must be used (i.e. a different value of 'p') to obtain the properly smoothed bicubic spline function defined on a rectangular grid to allow the SDG ridge crest to be mapped as a continuous location of the FCS that does not zigzag across the FCS. When the SDG algorithm was run on the raw data, the SDG obtained could not map the results with a simple plotting of each the largest z value in the row of the array being considered but zigzagged on either side of the actual FCS.$ 

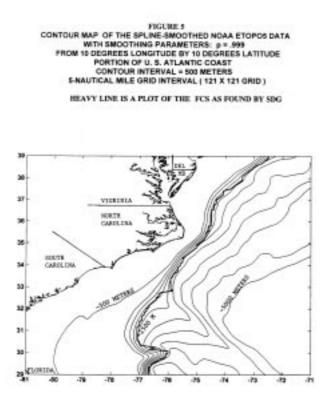
A more elaborate crest of the ridge tracing algorithm that discarded outliers to far from the last point found on the crest of the ridge is a possible solution. Another approach would be to smooth the data by decreasing the value of p until a reasonable FCS is found then iteratively increase the value of p until a credible FCS is found for the data with the higher information content.



**Figure 4** is a 3-D net display surface of the spline smoothed ETOPO5. A small amount of smoothing (p = .999) was applied. This was just enough smoothing to do the job. Just the right amount of smoothing is needed to eliminate enough noise to compute the second degree surface and also not omit the needed information content. The heavy black line in **figure 4** indicates the location of the FCS as determined by the SDG. Note the graphic location of the FCS to be: where the horizontal rows of diamonds in the 3-D net diagram are the largest and then where they begin to get smaller. This is a visual confirmation of the LOS legal definition of the FCS.i.e. where there is "the maximum change in the gradient."

This bi-cubic spline approach smoothes out the noise and represents the data as an explicit mathematical function. This function could be useful in many areas of oceanography.

**Figure 5** is a contour map of the same area as in the 3-D net diagram in **figure 4**. This method requires the original data set to have a rectangular grid. The heavy black line in **figure 5** indicates the location of the FCS as determined by the SDG, which is shown in **figure 6**.



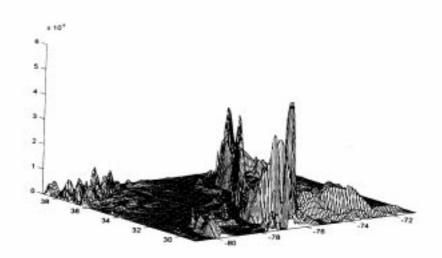
The notation in labeling the grid lines of the figures is in longitude and latitude. The original coordinate units for the ETOPO5 data are as follows: x: degrees longitude, y: degrees latitude, and z: meters measured below sea level. These coordinates should be measured on a sphere; however, in displaying the data and results of this paper, a flat surface is used without a map projection. One degree of latitude on the y-axis is approximately 60 nautical miles, or 5 minutes of latitude are approximately 5 nautical miles. When converting to nautical miles from degrees on the x or longitudinal axis, the farther the distance is from the equator, the smaller is a degree of longitude resulting with zero at the North Pole. Now that we have an explicit mathematical function that represents the ETOPO5 bathymetric data set with, if necessary, the noise removed, we proceed to apply the SDG. The approach of the SDG is to faithfully translate the UN legal definition of the FCS at each step to its equal event mathematical operation where the UN legal definition of the FCS can be implemented as the SDG algorithm on the bi-cubic spline bathymetric function that represents the digital ETOPO5 data.

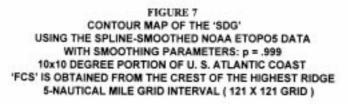
Other approaches previously used to compute the FCS use the surface of maximum curvature from the digital bathymetric data set; however, this is not a faithful mathematical implementation of the UN legal definition of the Law of the Sea.

To follow the lead of the legal description of FCS as cited above, one must proceed in the direction of the gradient from any given point of the digital bathymetric data set. The computational procedure generates the surface by computing the Rayleigh Quotient (second derivative matrix computation) in the normalized gradient direction on the smoothed bi-cubic spline function. The resulting surface is called the SDG. The location of the crest of the highest ridge of this surface is a good approximation to the determination of the FCS.

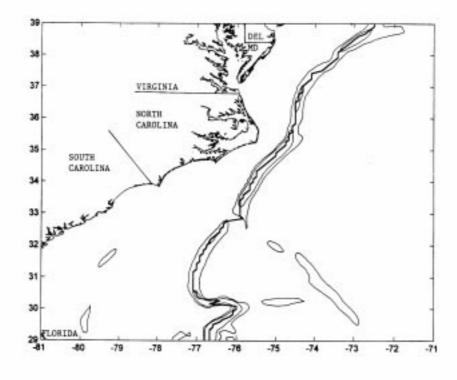
> FIGURE 6 3-D NET DISPLAY OF THE 'SDG' OF SPLINE-SMOOTHED NOAA ETOPOS DATA WITH p= .999 FROM 10 DEGREES LONGITUDE BY 10 DEGREES LATITUDE PORTION OF U. S. ATLANTIC COAST 5-NAUTICAL MILE GRID INTERVAL ( 121 X 121 GRID )

> > 'FCS' IS THE PEAK OF THE HIGHEST RIDGE





HEAVY LINE IS A PLOT OF THE 'FCS' AS FOUND BY SDG



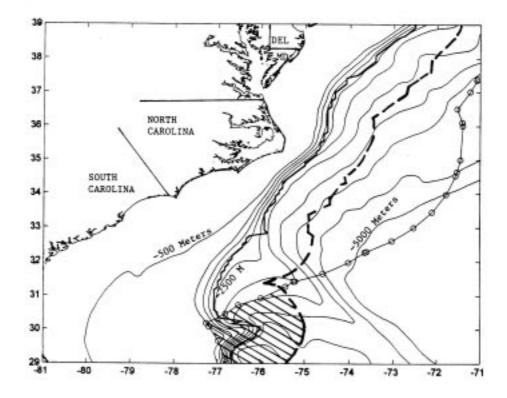
10

#### FIGURE 8

#### MAP SHOWING AREA WHERE FCS IS SEAWARD OF EEZ

CONTOUR MAP OF THE SPLINE-SMOOTHED NOAA ETOPO5 DATA WITH SMOOTHING PARAMETERS: p = .999 FROM 10 DEGREES LONGITUDE BY 10 DEGREES LATITUDE PORTION OF U. S. ATLANTIC COAST CONTOUR INTERVAL = 500 METERS 5-NAUTICAL MILE GRID INTERVAL (121 X 121 GRID)

HEAVY LINE IS A PLOT OF THE FCS AS FOUND BY SDG DASHED HEAVY LINE IS THE FCS PLUS 60 NAUTICAL MILES CIRCLED LINE IS EXTENT OF THE EXCLUSIVE ECONOMIC ZONE (EEZ)



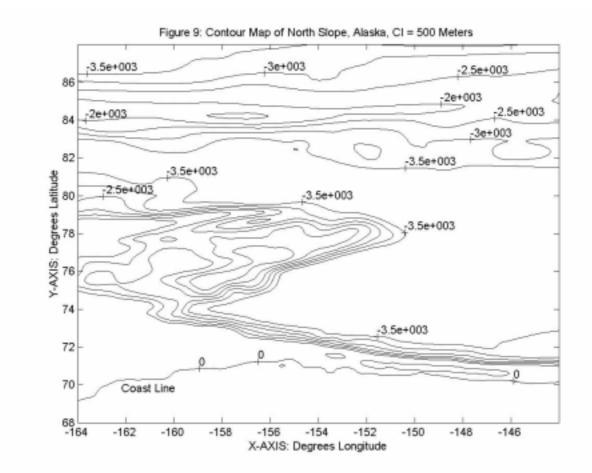
**Figure 8** is a map showing the bathymetric contours of the area. The map has a 500-meter contour interval. In **figure 8**, the FCS is represented by the solid heavy black line. The EEZ is represented by the circled black line. Note between 29 and 30 degrees North latitude the heavy black line, which is the FCS, is seaward of the 200-nautical mile EEZ. North of 30 degrees North latitude the EES (circled line) is always seaward of the FCS (solid black line). According to option 4 of the United Nation's LOS, the United States is entitled to the additional mineral rights between the FCS plus 60 miles line (heavy dashed line) and the EEZ line (circled line). These extended mineral rights are indicated by the hatched area between the solid and dashed heavy black lines in **figure 8**.

Although not shown on the map in **figure 8**, options 1 and 2 of United Nations LOS i.e. 1) 350 nautical miles from shore and 2) 100 nautical miles seaward of the - 2500 m isobath, respectively, given above would not limit the seaward extent of this possible U.S. claim as both lines would be well seaward of the heavy dashed line which represents the claim afforded to the United States by United Nation's LOS under option 3 in this particular case. Note the correlation between the FCS (black line) and the -2500 meter contour. (For a detailed mathematical presentation of the SDG see Bennett references 1 or 2.) It is clear that the accurate location of the FCS is important to any coastal country whose FCS beyond the 200-nautical mile EEZ.

Although some countries have not ratified the LOS including the United States, it should eventually. When this occurs, accurate digital location of the FCS will become increasingly more important to all coastal countries that wish to claim the entirety of their mineral rights. For any country to realize their full mineral rights they must first ratify the United Nations LOS. After a country ratifies the LOS it then has 10 years to submit their credentials for claims to extend their mineral rights.

## **EXAMPLE 2. NORTH SLOPE OF AKASKA**

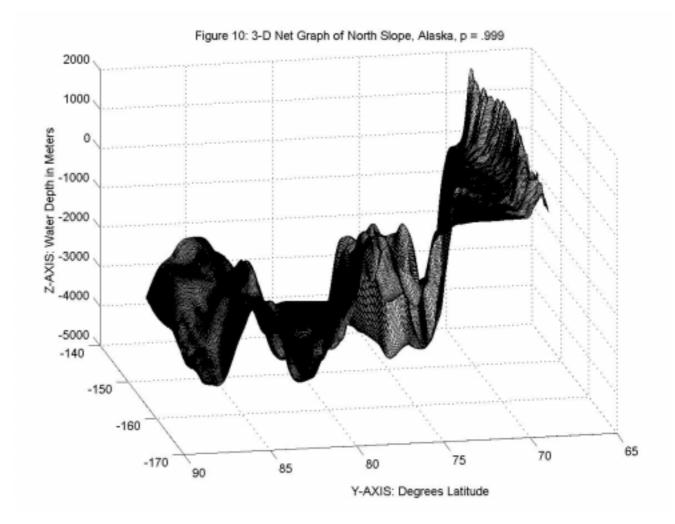
From what is a textbook example of the implementation of the SDG to the US Atlantic coast, we now turn to the North Slope of Alaska. It is not the classical continental shelf with a FCS that is readily classified by the UN legal definition of the FCS. The North Slope of Alaska has a sort of rolling hills type of sub-sea topography. The area we will consider is from -164 degrees to -146 degrees West longitude and 68 degrees to 86 degrees North latitude. Figure 9 is a contour map of this area under consideration.



The coastline is labeled with the zero, 0, contour.

Note the -2500 meter contours in the north part of the map. They would allow under option 2 above for expansion far beyond far beyond the 350 nautical Mile constraint of option 1, if the FCS could be extended that far north.

Note also the steep drop off along the coast in the eastern part of the map in Figure 9. Figure 10 is a 3-D Net Graph which shows the rolling hills sub-sea terrain north of the coast and suggests there could be several candidates for the FCS several hundred miles north of the first and primary FCS. This will be the case.



We will now compute the SDG for the North Slope of Alaska as displayed in the 3-D Net Graph in Figure 10. Note the 4 distinct ridges at 72-74(FCS-A), 76(FCS-B), 78(FCS-C), and 80 (FCS-D) degrees north latitude in the 3-D Net Graph. The crest of these four ridges FCS-A, FCS-B, FCS-C, and FCS-D are all candidates for locating a FCS.

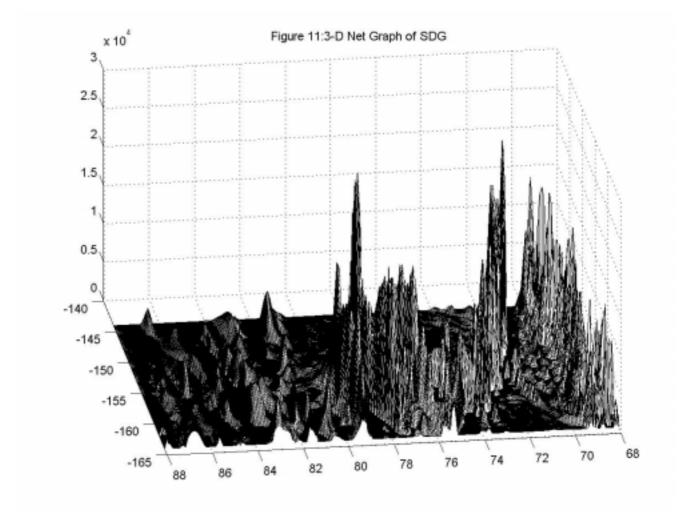
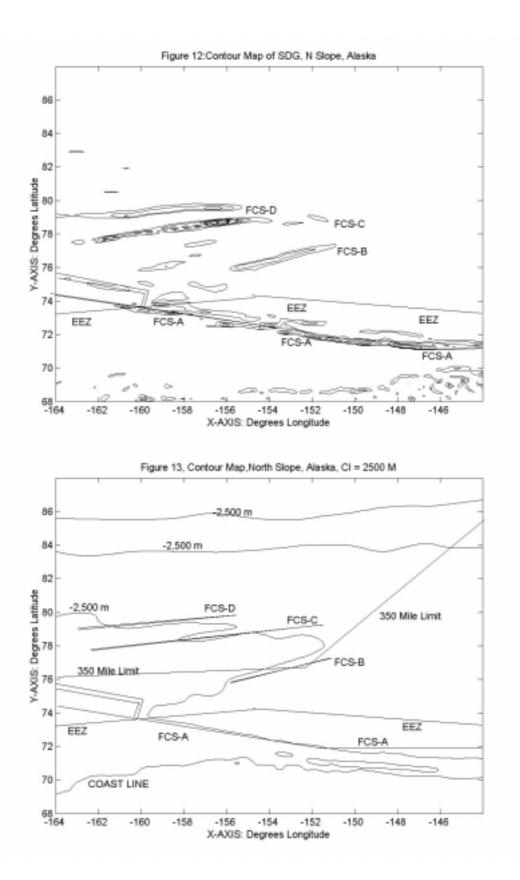


Figure 12 is a contour map of the SDG that is presented as a 3-D Net Graph in figure 11. Let us focus on FCS-A which starts at the eastern edge of the contour map in figure 12 at about 72 degrees latitude. It is located by the crest of the ridge labeled FCS-A. The approximate EEZ is indicated by the single line labeled EEZ. FDS-A is seaward of the 200 nautical miles from shore EEZ in the Eastern part of the map in figure 12. At about the (longitude degrees, latitude degrees) coordinate (-160,74) the EEZ and the FCS-A cross and west of this point the FCS-A is seaward of the EEZ. This invokes option 3 above since the FCS-A is seaward of the EEZ; hence, an additional 60 nautical miles is added to the location of the FCS-A. The additional mineral rights evoked by FCS-A are indicated on the maps in figures 12 and 13 by the area bounded by the double line (FCS-A plus 60 nautical miles), the EEZ boundary and the western border of the map. This area is allowed because it is landward of the 350-mile limit, option 1.



What is interesting about this North Slope of Alaska area is that there are in addition to FCS-A three other possible candidates for FCSs. They are FCS-B, FCS-C, and FCS-D. Although they are all seaward of the 350 Mile Limit they could be allowed by the –2,500 meter contours plus 100 miles by option 2 above.

### SUMMARY AND CONCLUSIONS:

1. The preprocessing of the NOAA's ETOPO5 worldwide bathymetric data set by bi-cubic spline smoothing provides an explicit mathematical function that can reduce the noise and retains most of the original information content of the data. This representation of the ETOPO5 data set as a mathematical function should provide valuable information about the world's sea floors to many areas of oceanography.

This data smoothing is usually requisite to obtaining good results in constructing any second derivative surface. Noisy data will always give poor results using gradient methods, because taking derivatives introduces angularity and magnifies the noise. There is a limit to the amount of smoothing that should be done. An estimate of the smoothing parameter "p" can be found as a function of the grid interval (See reference 1 page 8). This smoothing parameter "p" should not smooth out the original information content of the data. The optimal value of the smoothing parameter will vary depending on the data set. The explicit mathematical bi-cubic spline function used allows a possible display of the data at a finer grid than the grid on which it was originally received.

2. The SDG technique described in this paper can be implemented on any digital data set defined on a rectangular grid. If the data are from a triangulated grid, it must first be translated to a rectangular grid. I used the ETOPO5 data because it is a worldwide, public domain data set.

3. The SDG approach is an accurate, faithful mathematical modeling of the legal definition of the FCS; hence, I do not feel it can be improved on.

4. When the Surface of Maximum Curvature is used to compute the FCS it introduces spurious lobes that can obscure the location of the FCS. It also requires scaling of the data so that the x, y, and z-axes all are all of the same scale. After the results are obtained the axes need to be rescaled back so that the proper aspect ratio is retained.

5. The SDG does not have any spurious lobes and does not require scaling of the data to obtain accurate results.

6. The United Nations, legal definition of the FCS will not locate the FCS in all cases. In particular, when a cross-section perpendicular to the contours of the continental shelf is the arc of a circle, the legal definition will not yield a FCS. Because of the uniform gradient in this case, there is no maximum gradient at the base. When the FCS is formed under normal sedimentation conditions, this situation will be rare.

Where the FCS is sub-sea rolling hills such as in the North Slope of Alaska or very angular, an alternative definition to the current U. N.'s legal definition of the FCS might be required. For the U.S. Atlantic Coast ETOPO5 data set used here, the SDG located the FCS quite accurately 95 percent of the time for the entire coast.

7. With new technology allowing deeper drilling and mining, coastal countries are going to be more interested in the accurate location of their FCS when seaward of the EEZ.

8. It is hoped that the methods suggested in this paper and presented in Bennett references 1 or 2 will show how to utilize the information content of the NOAA ETOPO5 (or any other noisy rectangular digital data set) in many areas of oceanography and allow countries to use the SDG algorithm to compute their FCS where the U.N.'s LOS legal definition applies.

9. No country can claim their full mineral rights until they ratify the U.N. LOS. Once they ratify the LOS they have 10 years to present their claim to the UN Committee.

## **REFERENCES:**

1. Bennett, John, 1998, Mapping the Foot of the Continental Slope with Spline-Smoothed Data Using the Second Derivative in the Gradient Direction, US Department of the Interior, Minerals Management Service, OCS Report: MMS 97-0018

2. Bennett, John, 1998, Mapping the Foot of the Continental Slope with Spline-Smoothed Data Using the Second Derivative in the Gradient Direction, International Hydrographic Review, September, 1998

3. Bennett, John, 1999, Contrast of the 'Surface of Directed Gradient' with the 'Surface of Maximum Curvature to Compute the Foot of the Continental Slope. ABLOS Meeting, Monaco, September 7-10, 1999.

4. de Boor, Carl, 1978, A Practical Guide to Splines, Applied Mathematical Sciences #27, Springer-Verlag, New York, New York.

5.de Wet, Tertius, 1999, Issues and Scope of South Africa's Claim to a Continental Shelf TMT Technical Report # TV0010-9906-730.

6. Mira, Sjamsir (Editor), 1996, Geodetic Aspects of the Law of the Sea (GALOS), Second International Conference, Denpasar, Bali, Indonesia, 1-4 July 1996

7. United Nations Publication: Law of the Sea: Definition of the Continental Shelf, 1993 Sales # E.93.V.16, ISBN 92-1-133454-3.

8. United Nations Convention on the Law of the Sea: Fifth Meeting, New York, 24 July 2 August 1996. Commission on the Limits of the Continental Shelf: It's Function and the Scientific and Technical Needs in Assessing the Submission of a Coastal State.

9. United Nations Publication: Provisional Scientific and Technical Guidelines of the Commission on the Limits of the Continental Shelf, 1998, Commission on the Limits of the Continental Shelf, Fourth Session, September 4, 1998.

10. Vanicek, Petr; Wells, David E.; Hou T., 1994, Determination of the Foot of the Continental Slope, Contract Report, Geological Survey of Canada, Atlantic Geoscience Centre, Bedford Institute of Oceanography, Dartmouth, N. S. B3A4A2, SCC# OSC939-3-R207/01-OSC.

11. Ou, Ziqiang ;Vanicek, Petr, Automatic Tracing of the Foot of the Continental Slope, Marine Geodesy, volume 19, Number 2, 1996