

# AN ALGORITHMIC SOLUTION TO THE RANDOMNESS OF EQUITABLE BOUNDARY LINES

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## 1. Introduction

Maritime delimitation usually starts with the calculation of an equidistance line, which is often corrected for special circumstances. The correction process aims to produce a line that is “equitable”, a concept open to various interpretations. While the equidistance line is calculated by a technical expert, an equitable line is the result of a negotiation process. Technical support is, in that case, only necessary to argue that a certain line is equitable indeed. Of course, the ideal situation is when the negotiations result in an agreement that the equidistance line is equitable as well.

Boundary negotiations sometimes consist of the proposal and defense of a large series of boundary candidates, which are claimed to be equitable by one of the States. This way, a large set of potential boundary lines is built up, a situation that is hard to control and visualize. An example is the presence of small islands in front of the coast, that have a disproportionate effect on the equidistance line. A series of more or less random corrections of the equidistance line could mitigate the disproportionate effect.

Algorithmic methods for the calculation of an equitable boundary line have the potential to improve and speed up such negotiations. Instead of selecting the most equitable candidate out of a potentially very large set of boundary candidates, agreement could be reached over a method in combination with assigning a value to a small set of methodical parameters. The result is a single boundary line, firmly based on the geographic configuration. Those methodical parameters reflect weighting factors of points on the UNCLOS baseline, based on the character of those base points. Examples are islands, rocks, low tide elevations, and permanent harbor works.

We propose an algorithmic method that is more flexible than equidistance, yet the output is of the same simplicity: a series of turning points that should be connected by lines that are agreed to be straight by the parties. Straight lines are e.g. loxodromes or geodesics on the selected ellipsoid. The proposed method is a simplification of the equiratio method.

We provide an overview of methods and their relations in Section 2, and describe the equiratio method and its proposed simplification in Section 3. Section 4 contains an example, Section 5 a discussion on the insights that this example gives, and Section 6 draws conclusions based on the discussion.

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<sup>1</sup> The views expressed in this paper represent the opinion of the author. They do not necessarily coincide with any position of the Netherlands government.

## 2. Methods for the construction of boundary lines

The distances covered by maritime boundaries often are so large that calculations need to be done on an ellipsoid, instead of in a planar projection, to prevent inaccuracies. Mathematical solutions for the calculation of coordinates, distances and azimuths on the ellipsoidal surface are given by [1]. These solutions form the geodetic basis of the algorithmic methods for maritime boundary calculation. This is illustrated by [2] for the calculation of the equidistance line.

The ABLOS Manual on Technical Aspects of the Law of the Sea [3] identifies several algorithmic methods to correct an equidistance line. They are the partial effect line; notional base points; movement of the equidistance line; the equiratio line; the bisection of the general directions of the two coasts.

The partial-effect line assumes that two equidistance lines are already available: a full-effect line that includes all relevant points on the baseline, and a no-effect line that does not include specific points. The half-effect line runs exactly in between those two lines, or – in other words – is the equidistance line between the full-effect and the no-effect line. In an iterative way, other partial-effect lines can be constructed: a quarter-effect line, an eighth-effect line, et cetera. Half-effect is described in detail by [4].

Notional base points are points on an imaginary baseline, included in the calculation of an equidistance line. The transfer of a point in the direction of the other State increases its weight, and the transfer in the opposite direction reduces the weight, or even makes it irrelevant. Notional base points and movement of the equidistance line share the same subjectivity: how large should a shift be, and in which direction?

The equidistance line is a special case of the equiratio line. This method applies weights to all base points, resulting in a line at a corresponding distance ratio between the base points. In case all weights are equal, the equidistance line is reproduced. Equiratio is discussed in more detail in the next Section.

If the two baselines are both reduced to a line with a constant direction, the result of the equidistance method is a bisector line. The constant direction of that bisector corresponds to the average of the directions of the two baselines in the point where they intersect. If the bisector method is applied to a series of straight baselines, the resulting algorithm corresponds to the half-effect algorithm.

The bisector line and the partial-effect lines are special cases of the equidistance line, and the equidistance line is in turn a special case of the equiratio line. These methods together form a first category of purely algorithmic methods. Notional base points change the input of the equidistance procedure, not the method. And, finally, also the movement of the equidistance line is evidently linked to the equidistance line. These two methods form a second category of semi-

algorithmic methods. Therefore, the equiratio method includes the methods of the first category, and is the core of the methods of the second category.

### **3. The equiratio method, and its simplification**

We identify equiratio as the most flexible, but also as the most complicated method. It was developed by Langeraar [5,6,7,8] and has – to our knowledge – not yet been applied in a maritime boundary delimitation process. The input consists of a limited set of base points that are assumed to describe the baseline to a sufficient degree, and the weight of each base point.

A line between two base points with a different ratio curves towards the base point with the smaller ratio. This curve is circular. Equiratio lines are created by calculating circle sectors for sets of one point on one baseline, and the other point on the other baseline. Where the curves intersect, a turning point is created. In the case of a small island State, represented by a single base point, in front of a State with a baseline of constant direction, the equiratio line approaches a parabola if the number of selected points on the baseline approaches infinity. Langeraar illustrated the viability of the equiratio method by approximating several maritime boundary lines that are considered equitable but that are not equidistant. The most known of these are the North Sea continental shelf cases [9].

Although the equiratio method provides a series of coordinates of turning points, the curves in the connecting lines severely complicate the description of a boundary line in a legal document. In spite of this, the present state of information technology allows for the description of such a boundary, and for the approximation of a curve by a large series of coordinates. Curved lines are e.g. part of a recent judgment of the International Court of Justice [10]. However, a method that results in turning points that could be connected by an agreed straight line is still far more desirable.

If we use the equiratio principle as an adaptation to the turning points of the equidistance line only, we maintain the simple representation of this equidistance line, while creating an opportunity to change its outcome in an automated way. This allows for equitable solutions that are not created by changing the location of turning points manually, or in a semi-automatic way (the second category mentioned in the previous Section). It turns out that minor modifications to the equidistance algorithm of [2] create such an algorithm. We call this algorithm simplified equiratio (SER).

Each equidistant turning point is the intersection of a line in between two base points, and another line that has one base point in common with the first line. The turning point is calculated as the point with equal distance to all three these base points, using a “three-point algorithm”. Instead of keeping these three distances equal, we could vary them according to a previously determined ratio. The position of the turning point changes correspondingly.

The “three-point algorithm” is an iterative procedure that starts with a planar approximation. The only price that we have to pay to allow for an equiratio solution is the complexity of the planar approximation, turning into an iterative procedure as well. As planar calculations tend to be much faster than ellipsoidal calculations, the extra calculation time is limited.

#### **4. An example**

In Figure 1, four lines are presented that have been calculated using equidistance, in a realistic but made-up sea area full of small islands. Also, there are two islands of medium size and the three areas of mainland. One of the mainland areas belongs to one State, the other one to another State, and the third mainland area is shared between the two States. Therefore, we created a combination of adjacent and opposite coasts.

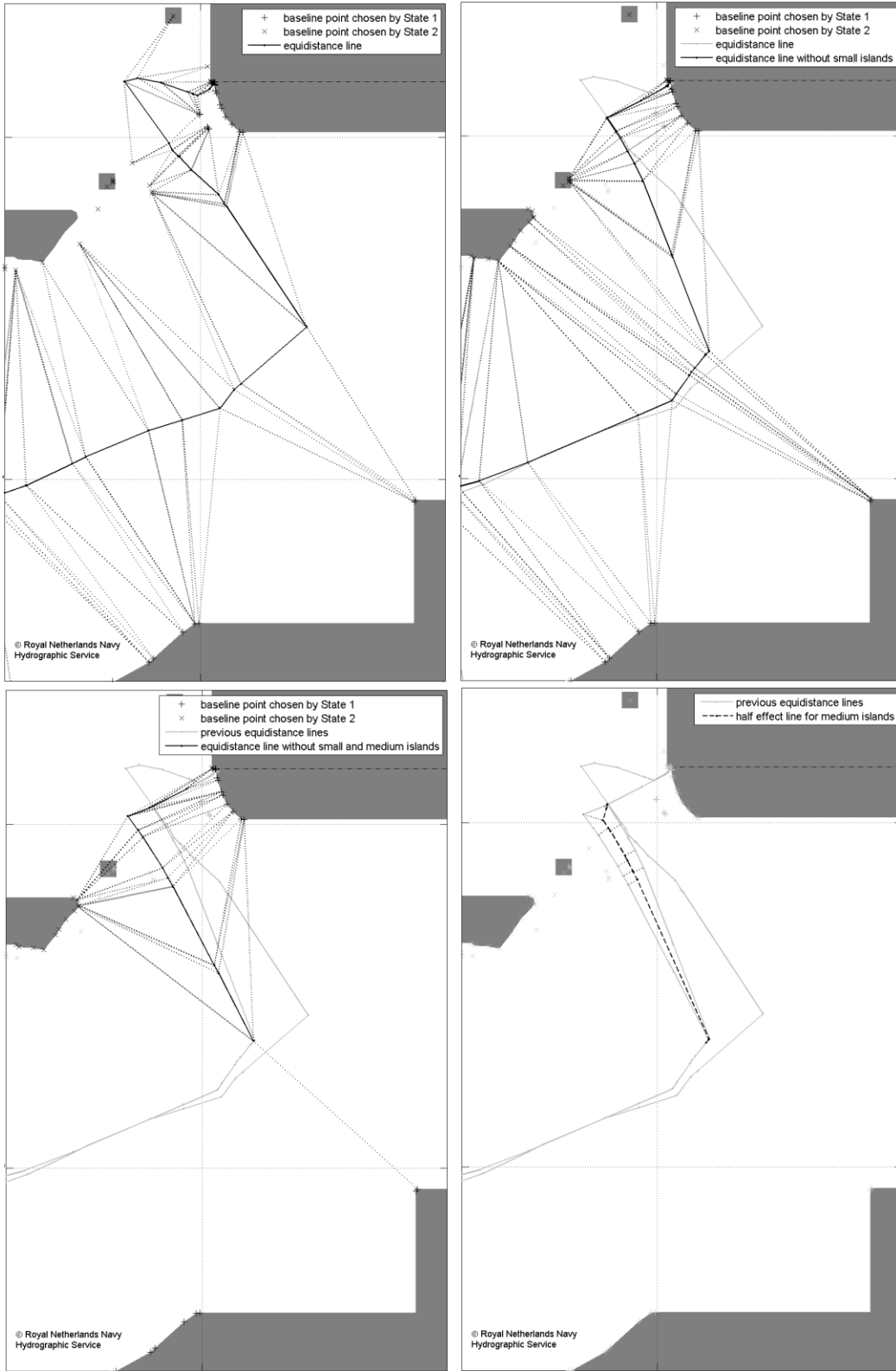
The equidistance line between these two States consists of three different parts, see the top left map. The first part runs West from the shared mainland. After a sharp turn, it runs in a Southern direction between the small islands of the two States, and after a second sharp turn it runs West again, in between the two other areas of mainland and a few additional small islands.

The influence of the small islands becomes obvious when we calculate a second equidistance line, ignoring the small islands, in the top right map. The two sharp turns were clearly caused by two small islands, one of each State. The medium-sized island that was relevant at first, is no longer used, and the medium-sized island that was sheltered by the small islands now has become relevant.

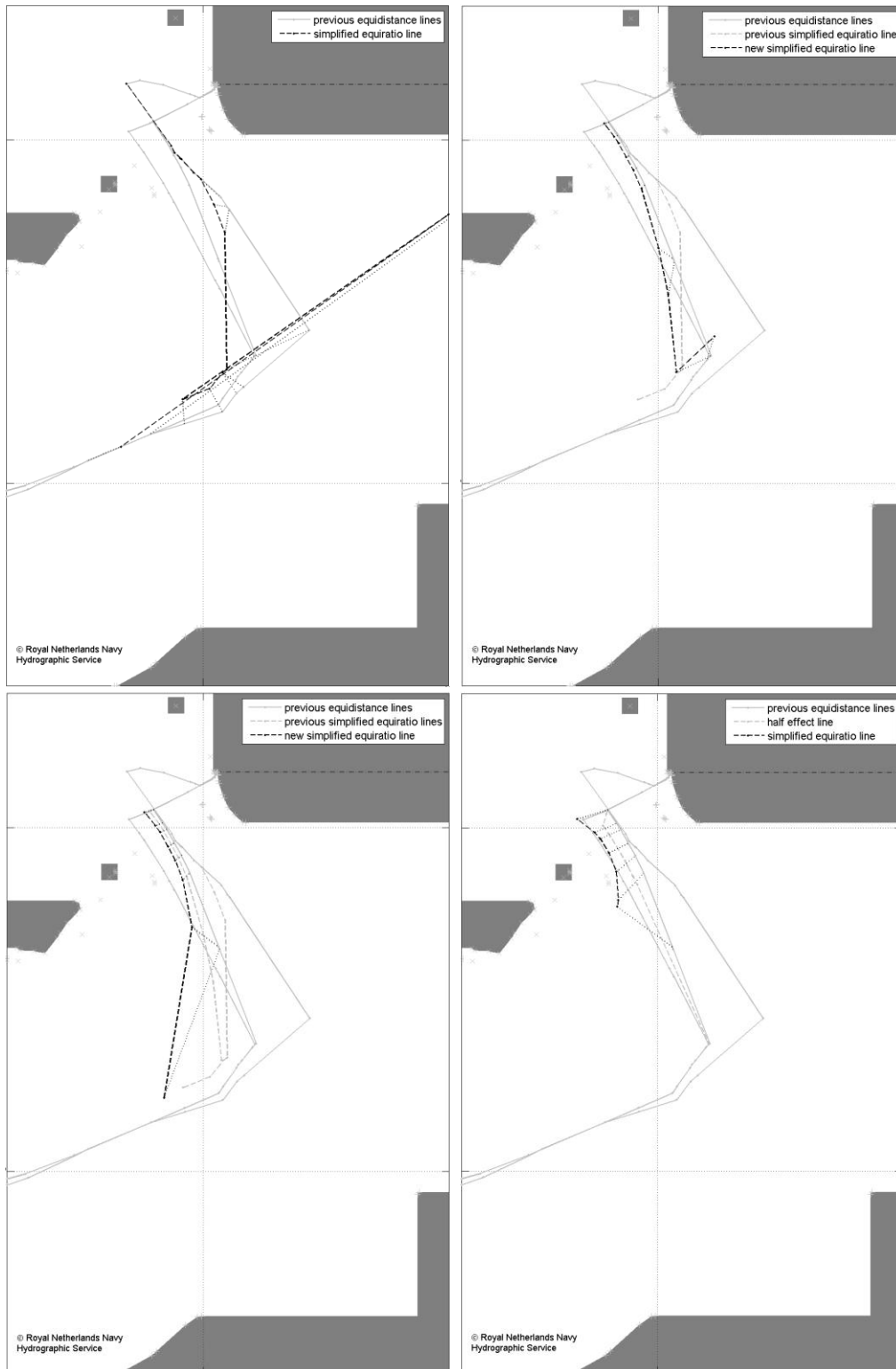
We might argue that this still is unequitable, and reduce the influence of the medium-size island. To do this, we apply the half-effect method to this island. First, we calculate a third equidistance line without the base points on the medium-sized island, shown in the bottom left map of Figure 1. The line that runs in between the second and the third equidistance line is the half-effect line. This line is give in the bottom right map of Figure 1.

It took us four steps to arrive at this solution. If this line still is not accepted as equitable by one of the parties, our options are limited to the semi-algorithmic methods of category two, introducing subjectivity. The regularly applied fully algorithmic methods of category one cannot help out anymore, as even the calculation of a quarter-effect line would have a very limited influence only.

The only other fully algorithmic method is equiratio. In its simplified version, we start with the equidistance line again, as is common practice. During the negotiations, the two parties can now propose and counter-propose different ratios, and interpret the shifted positions of the turning points until they are satisfied. Moreover, the process could consist of two steps only: one equidistance calculation and one simplified equiratio calculation.



**Figure 1:** Equidistance line between all basepoints (top left); without small islands (top right); without small and medium islands (bottom left); with half effect of medium islands (bottom right).



**Figure 2:** Simplified equiratio line with a ratio of 0.8 for small and 0.9 for medium size islands (top left); without small islands and 0.9 for medium size islands (top right); idem, 0.8 for medium size islands (bottom left); idem, 0.5 for medium size islands (bottom right).

Let us first try a ratio of 0.9 for the medium sized islands, and a ratio of 0.8 for the small islands. The results are shown in the top left map of Figure 2. Each turning point has shifted according to its dotted line. For reasons explained below, we start at the first sharp turn. The equidistance line does not show any change where the line runs in between small islands, as all the base points have the same 0.8 ratio. Suddenly, the new line shows erratic behavior, and we have to reject this solution. A comparison with the original base points of the erratic point explains then situation: where two points of one baseline lie relatively close together, but receive a different ratio, the turning point needs to be shifted over a large distance in order to arrive at the required ratio. In this case, the 0.8 ratio of a small island close to the mainland, in combination with the mainland ratio of 1.0, creates such an erratic situation. In fact, the same situation applies to the first part of the equidistance line, where the baseline of one State has relevant points on the mainland (1.0 ratio), a small island (0.8 ratio), and a medium sized island ( 0.9 ratio), all in each other's direct vicinity.

We create a less challenging situation by starting with the second equidistance line, without the small islands. In this case, there are only mainland base points that receive a 1.0 ratio and a few additional base points that receive a 0.9 ratio. The resulting equiratio line for the second part of the boundary is very usable, especially because it reduces the second sharp turn to a gradual change. Unsurprisingly, when we have to make the combination between the 0.9 ratio base point and the 1.0 base point on the mainland, the result again is erratic.

We ignore this erratic point, and apply the method to the middle part of the equidistance line only. Now, it is possible to find out what happens if we change the ratios during the negotiation process. A ratio of 0.8 for the medium-sized island results in the bottom left map of Figure 2, and the extreme ratio of 0.5 in the bottom right map. From this last map, it is also clear that the application of a 0.5 ratio to a group of base points is a very different solution than assigning half effect to them.

## **5. Discussion**

The equiratio method has the potential to provide firm algorithmic solutions to complicated boundary negotiation processes, while the application of ratios allows for an outcome based on offers and counter-offers. The main disadvantage of the equiratio method is that its output does not consist of a limited set of straight line segments. Instead, every segment is curved, which makes the resulting boundary a complicated line that is not easily described in e.g. a treaty.

The simplified equiratio (SER) method reduces the complexity of the output, but this brings a second disadvantage to light. Close base points should have similar ratios, whether they are located on the mainland or not. Probably, this is the reason that Langeraar's examples had equal ratios for each State. This problem is circumvented by doing separate calculations for each part of an equidistance line. In our example, we would split up the equidistance line in three parts, and only use SER for the middle part.

Although SER is fully algorithmic, it is clearly not fully automatic. The application has to come with awareness of the erratic behavior, and with knowledge on how to circumvent such behavior. SER helps the technical expert in boundary negotiation processes because of its flexibility, but it also demands a high level of understanding of the methodical aspects. It strongly depends on the specific situation if SER is a blessing or a burden.

## **6. Conclusion**

Contentious issues in maritime boundary delimitation processes are alleviated using advanced algorithmic methods, especially in case of islands of different sizes. We propose to add the simplified equiratio method (SER) to the known set of methods, because it combines the flexibility of the equiratio method with the output characteristics of the equidistance method. However, it is not evident that the results are free from erratic turning points.

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