The 4th Tidal and Water Level Working Group Meeting (May 8-10, 2012)

Examining classical tidal harmonic prediction algorithms

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Motivation

As shown in Stephen's and Christopher's presentations, there are slightly different prediction results according to use of harmonic analysis and prediction programs.

Why ?

Motivation

Bill's question

I transformed the list into ASCII form and write a program to compute the speed of each constituents based on the XDO value.
Basically it sums the products of the Doodson number multiplied with the speed of that argument. Speed of individual Doodson arguments are taken from papers such as Simon et al (1994) or Doodson (1921) since the Standard Constituent List did not list out these numbers. I have tried out many sets of possible speeds (of the arguments), and I matched the speed of nearly all of the constituents, but not 6 out of the 7 in the ninth-diurnal section. I have to conclude that there may be computation error on the speed of these 6 constituents: 3MNO9, 2M2NK9, 2(MN)K9, 3MNK9, 4MK9, 3MSK9.

Here comes an excerpt of my computation result, where the columns are:
1: Speed, as in the Standard Constituent List
2: (Speed derived from the XDO by my program) - (Speed on the Standard Constituent List)
3~9: Doodson numbers of the constituent
10: Nodal correction flag, as in the Standard Constituent List
11: Name of the constituent, as in the Standard Constituent List

118.521867	-1.86e-07	8	5	-4	1	0	0	0	x	2SKN8
118.594720	+4.67e-07	8	5	-2	-1	0	0	2	x	MSKL8
118.984104	+2.4e-07	8	6	-6	0	0	0	0	x	3SM8
119.066242	-4.8e-07	8	6	-4	0	0	0	0	x	2SMK8
120.000000	+0	8	8	-8	0	0	0	0	x	S8
129.335076	+1.86e-06	9	-2	0	1	0	0	-1	x	3MNO9
129.888738	-1.81e-06	9	-1	0	2	0	0	1	x	2M2NK9
129.888738	-1.81e-06	9	-1	0	2	0	0	1	x	2(MN)K9
130.387400	+4.42e-07	9	0	-1	0	0	0	0	x	MA9
130.433113	-2.1e-06	9	0	0	1	0	0	1	x	3MNK9
130.977488	-2.4e-06	9	1	0	0	0	0	1	x	4MK9
131.993383	-1.64e-06	9	3	-2	0	0	0	1	x	3MSK9
143.360251	-2.64e-07	10	-3	2	1	0	0	0	x	5MNS10
143.831772	-2.11e-07	10	-2	0	2	0	0	0	x	3M2N10
143.904625	+4.42e-07	10	-2	2	0	0	0	0	x	6MS10
143.913909	+6.9e-08	10	-2	2	2	0	0	0	x	3M2NKS10
144.294009	+2.16e-07	10	-1	-2	1	0	0	0	x	4MSNK10

Fundamental astronomical variables

Byun & Cho (2009)

Table 2
Periods and angular speeds of the six fundamental astronomical variables used in tidal prediction and analysis.

Astronomical variables	Period		Angular speed (deg. h^{-1})
T	Mean lunar day (T_p)	1.03505 solar days	14.49205212
s	Sidereal month (s_p)	27.321582 solar days	0.5490165207
h	Tropical year (h_p)	365.242199 solar days	0.0410686393
p	Moon's perigee (p_p)	8.847 years	0.00464396
N	Moon's ascending node (N_p)	18.613 years	0.0022064
p'	Perihelion (p'_p)	20.940 years	0.0000019617

Table 3
Results of harmonic analysis of 12 major tidal constituents using 1 year of sea elevation data from Incheon Tidal Station 2007, and the constituent angular speeds (ω).

Constituents	Composition	ω (° h^{-1})	NMC[u, f]	Astronomical origin	Amp. (cm)	Phase-lag (°)
M_2	$2T_p$	28.9841042	$f(N)$	Moon	287	129
S_2	$2T_p + 2s_p - 2h_p$	30.0000000	–	Sun	117	187
N_2	$2T_p - s_p$	28.4397295	$f(N)$	Moon	50	108
K_2	$2T_p + 2s_p$	30.0821373	$f(N)$	Moon and Sun	33	182
L_2	$2T_p + s_p - p_p$	29.5284789	$f(p, N)$	Moon	12	121
Nu_2	$2T_p - s_p + 2h_p - p_p$	28.5125831	$f(N)$	Moon	11	101
Mu_2	$2T_p - 2s_p + 2h_p$	27.9682084	$f(N)$	Moon	12	176
K_1	$T_p + s_p$	15.0410686	$f(N)$	Moon and Sun	40	303
O_1	$T_p - s_p$	13.9430356	$f(N)$	Moon	29	264
P_1	$T_p + s_p - 2h_p$	14.9589314	–	Sun	11	295
M_4	$4T_p$	57.9682084	$f(N)$	–	6	127
MS_4	$4T_p + 2s_p - 2h_p$	58.9841042	$f(N)$	–	6	190

Note that phase-lags are referenced to 135°E, NMC is nodal modulation correction and N and p indicate the Moon's ascending node (18.613 years) and its perigee (8.847 years), respectively. The angular speeds of the astronomical variables (T_p, s_p, h_p, p_p) are listed in Table 2.

Bill's 'readlist.c'

```
  FILE *infile;
  int lookUp[255];
  int XDOint[7];
// double XDOSpeed[7] = {14.49205212, 0.54901651990, 0.04106863988, 0.00464181350,-
0.00220640711, 0.00000196146,0}; /* "Current" best value -- 6 fails */
  double XDOSpeed[7] = {14.49205212018, 0.54901651973, 0.04106863991,
0.00464181341,-0.00220640687, 0.00000196151,0}; /* Simon et al (1994) -- 6 fails */
// double XDOSpeed[7] = {14.49205211, 0.54901653, 0.041068638, 0.004641836,-
0.002206413, 0.000001961,0}; /* Doodson (1921) -- 19 fails */
  char c; int i;
```

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His problem is likely to relate angular speeds of astronomical variables.

Suggestion

TASK - Tidal Analysis Software Kit - 2000 ; December 99

Name	Speed	(deg/hr)			
			58	2MK6	88.0503457
1	Sa	0.0410686	59	2SM6	88.9841042
2	Ssa	0.0821373	60	MSK6	89.0662415
3	Mm	0.5443747	61	2MN2S2	26.4079379
4	Msf	1.0158958	62	3M(SK)2	26.8701754

SHOM & UKHO

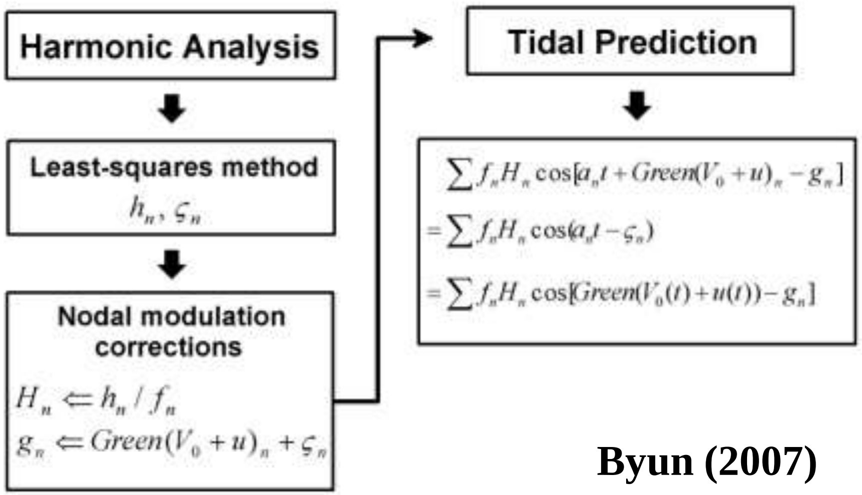
Name	Speed (Deg/hr)	XDO		Nodal Correction
		Numerical	Alphabetical	
Long Term Constituents				
Zo	0.000 000	0 555 555	Z ZZZ ZZZ	z
Sa	0.041 067	0 565 545	Z ZAZ ZYZ	z
Sa	0.041 069	0 565 555	Z ZAZ ZZZ	z
Ssa	0.082 137	0 575 555	Z ZBZ ZZZ	z
Sta	0.123 204	0 585 544	Z ZCZ ZYY	x
MSm	0.471 521	0 636 555	Z AXA ZZZ	x

➔ Need to write down to Seven places of decimals like Task-2k.

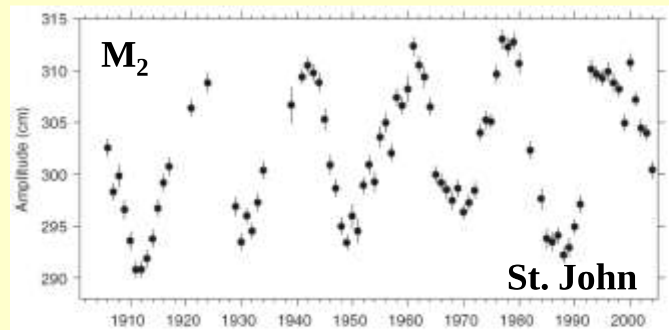
Questions?

- What are the key tidal prediction factors in harmonic tidal prediction methods?
- Are there any accuracy difference among formulas for five astronomical variables commonly used in tidal prediction?
- How often the NMC needs to be conducted in tidal prediction simulations to obtain relatively accurate predictions?

Tidal prediction



Byun (2007)



Ray (2006)
CSR 26

$$h(t) = \sum_{n=1}^{N} f_n(t) H_n \cos(V_{0n}(t) + u_n(t) - g_n)$$

$$\because V_{0n}(t) = V_{0n}(t_o) + \omega_n t$$

$$h(t) = \sum_{n=1}^{N} f_n(t) H_n \cos(V_{0n}(t_o) + \omega_n t + u_n(t) - g_n)$$

Astronomical argument (Vo)

```
      SUBROUTINE VSET (S,H,P,P1,V)
      REAL*8 S,H,P
      DIMENSION V(120)
C     V 'S  ARE COMPUTED HERE.
C     V'S COMPUTED IN DEGREES.
      V(1) = H
      V(2) = H+H
      V(3) = S-P
      V(4)= S+S -H-H
      V(5) = 2*S
      V(6) =-4*S+H +2*P+ 270.0
      V(7) =-4*S+3*H +270.0
      V(8) =-3*S+H+P + 270.0
      V(9) =-3*S+3*H-P+270.0
      V(10) =-2*S+H + 270.0
      V(11) = 3.0*H-2.0*S + 90.0
      V(12)=-S+H+90.0
      V(13)=-S+3*H-P+ 90.0
```

Astronomical variables	Period	Angular speed (deg. h^{-1})
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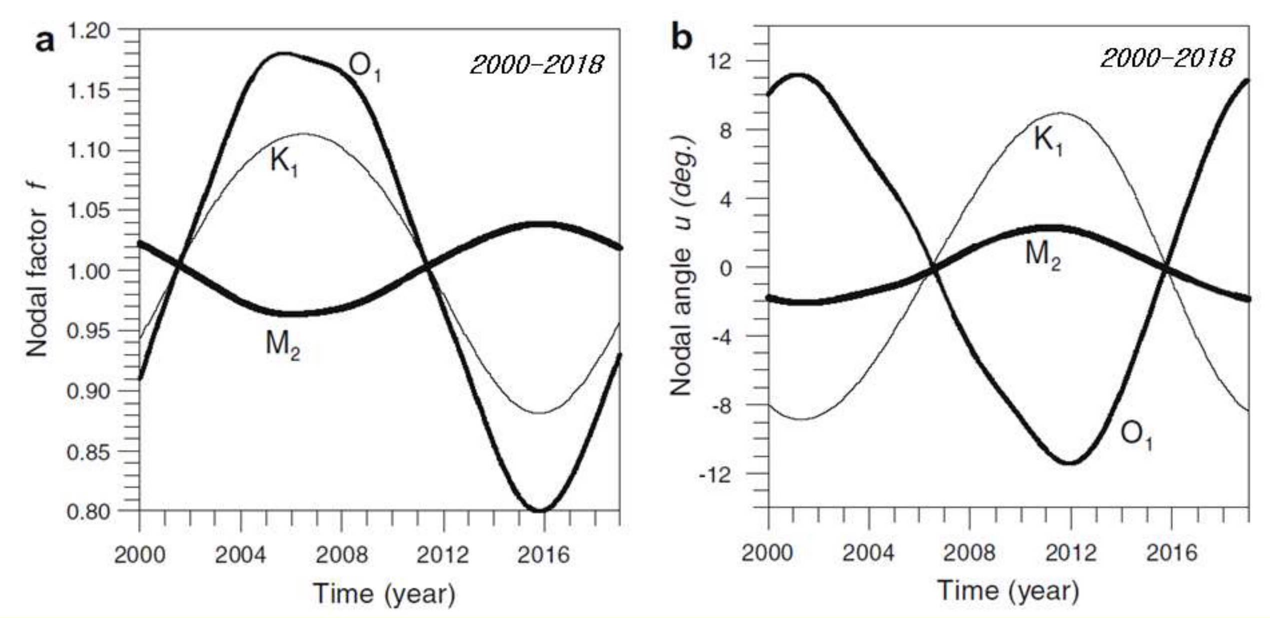
Astronomical arguments of tidal constituents are calculated from values of astronomical variables.

Modulation of [*f*, *u*]



f_{M2} : 0.96 ~ 1.04
u_{M2} : -2.1º ~ 2.3º

f_{K1} : 0.88 ~ 1.11
u_{K1} : -8.9º ~ 9.0º

f_{O1} : 0.80 ~ 1.18
u_{O1} : -11.4º ~ 11.2º

Incheon

Constituents	Amp. (cm)	Phase-lag (°)
M_2	287	129
S_2	117	187
N_2	50	108
K_2	33	182
L_2	12	121
Nu_2	11	101
Mu_2	12	176
K_1	40	303
O_1	29	264
P_1	11	295
M_4	6	127
MS_4	6	190

M_2
275.5~298.5 cm ➔ 23 cm
126.9~131.3º ➔ 4.4º

K_1
35.2~44.4 cm ➔ 9.2 cm
294.1º~312º ➔ 17.9º

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Note that phase-lags are referenced to 135°E, NMC is nodal modulation correction and N and p indicate the Moon's ascending node (18.613 years) and its perigee (8.847 years), respectively. The angular speeds of the astronomical variables (T_p, s_p, h_p, p_p) are listed in Table 2.

Formulas for astronomical variables

Table

Formulas for five astronomical variables commonly used in tidal prediction programs (i.e. IOS and Task-2000) and models.

Case	Formula (unit: deg.) $\phi_i(t) = \phi_i(t_0) + \phi_i(t - t_0)$	$\phi_i(t_0)$	Reference origin (t_o)	Reference
Type 1 [IOS]	$s = s_o + 13.1763965268$ da $- 0.000085$ da $10^{-8} + 0.000000039$ da 10^{-12} $h = h_o + 0.9856473354$ da $+ 0.00002267$ da^{-8} $p = p_o + 0.1114040803$ da $- 0.0007739$ da$^{-8} - 0.00000026$ da^{-12} $N = N_o - 0.0529539222$ da $+ 0.0001557$ da$^{-8} + 0.00000005$ da^{-12} $p' = p'_o + 0.0000470684$ da $+ 0.0000339$ da$^{-8} + 0.00000007$ da^{-12}	$s_o = 270.434164$ $h_o = 279.696678$ $p_o = 334.329556$ $N_o = 259.183275$ $p'_o = 281.220833$	1200 GMT Dec 31, 1899	Foreman (1996)
Type 2 [Task-2000]	$s = s_o + 129.38481$ (Y-1900) $+ 13.17639$ DL $h = h_o - 0.23872$ (Y-1900) $+ 0.98565$ DL $p = p_o + 40.66249$ (Y-1900) $+ 0.11140$ DL $N = N_o - 19.32818$ (Y-1900) $- 0.05295$ DL $p' = p'_o + 0.017192$ (Y-1900)	$s_o = 277.0247$ $h_o = 280.1895$ $p_o = 334.3853$ $N_o = 259.1568$ $p'_o = 281.2209$	0000 GMT Jan 1, 1900	Bell et al. (1999)
Type 3	$s = s_o + 129.38482$ (Y-1900) $+ 13.1763968$ DL $+ 0.54901653$ H $h = h_o + 359.76128$ (Y-1900) $+ 0.9856473$ DL $+ 0.04106864$ H $p = p_o + 40.66247$ (Y-1900) $+ 0.1114040$ DL $+ 0.00464183$ H $N = N_o - 19.32819$ (Y-1900) $- 0.0529539$ DL $- 0.00220641$ H $p' = p'_o + 0.01718$ (Y-1900) $+ 0.0000471$ DL $+ 0.00000196$ H	$s_o = 277.026$ $h_o = 280.190$ $p_o = 334.384$ $N_o = 259.156$ $p'_o = 281.221$	0000 GMT Jan 1, 1900	Table 1 of Schureman (1958)
Type 4	$s = s_o + 481267.89$ JC $+ 0.0011$ JC^2 $h = h_o + 36000.77$ JC $+ 0.0003$ JC^2 $p = p_o + 4069.04$ JC $+ 0.0103$ JC^2 $N = N_o - 1934.14$ JC $+ 0.0021$ JC^2 $p' = p'_o + 1.72$ JC $+ 0.0005$ JC^2	$s_o = 277.02$ $h_o = 280.19$ $p_o = 334.39$ $N_o = 259.16$ $p'_o = 281.22$	0000 GMT Jan 1, 1900	Pugh (1987)

Note that da = the current DN–693961.5 (noon 31/12/1899) where DN is the day number based on the Gregorian calendar date from 1200 GMT on Dec 31 1899, DN = 693961. DL = INT[(Y–1901)/4] + D – 1 for the years 1900–2099, DL = –{INT[abs(Y–1900)/4]} + D – 1 for the years 1801–1899, Y is the year and D is the day number in the year. H is the hour and JC = {365(Y–1900) + DL}/36525 which is the number of the Julian Century (36525 mean solar days).

Calculated results of astronomical variables

Time	Case	Fundamental astronomical variables (°)					M$_2$ tide		
		s	*h*	*p*	*N*	*p'*	*Vo* (°)	*u* (°)	*f*
00:00 1/1/1900	Type 1	277.022	280.190	334.385	259.157	281.221	6.334	2.102	1.007
	Type 2	277.025	280.190	334.385	259.157	281.221	6.330	2.102	1.007
	Type 3	277.026	280.190	334.384	259.156	281.221	6.328	2.102	1.007
	Type 4	277.020	280.190	334.390	259.160	281.220	6.340	2.102	1.007
00:00 1/1/2000	Type 1	211.728	279.973	83.298	125.070	282.940	136.490	358.249	1.022
	Type 2	211.739	279.973	83.308	125.068	282.940	136.468	358.248	1.022
	Type 3	211.742	279.977	83.305	125.066	282.940	136.469	358.248	1.022
	Type 4	211.729	279.975	83.329	125.075	282.940	136.491	358.249	1.022
00:00 1/1/2100	Type 1	159.608	280.743	192.301	350.934	284.661	242.270	0.337	0.964
	Type 2	159.631	280.742	192.343	350.926	284.659	242.225	0.337	0.964
	Type 3	159.633	280.742	192.337	350.923	284.659	242.221	0.338	0.964
	Type 4	159.579	280.743	192.400	350.941	284.662	242.630	0.337	0.964

Case	Reference
Type 1 [IOS]	Foreman (1996)
Type 2 [Task-2000]	Bell et al. (1999)
Type 3	Table 1 of Schureman (1958)
Type 4	Pugh (1987)

Astronomical variables	Period	
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The differences between results calculated from formulas are insignificant.

Tidal analysis & prediction packages

Task-2000 package

- Proudman Oceanographic Laboratory, POL
- tira.f (harmonic analysis), marie.f (prediction)
- **NMC update period : every 3 days**

IOS tidal package

- Institute of Ocean Sciences, IOS (Dr. Mike Foreman)
- Tide1_r2.f (harmonic analysis), Tide4_r2.f (prediction)
- **NMC update period : every 15 days**

※ T_TIDE (Pawlowicz et al., 2002) - MATLAB version

Questions ?

- What are the key tidal prediction parameters in harmonic tidal prediction methods?
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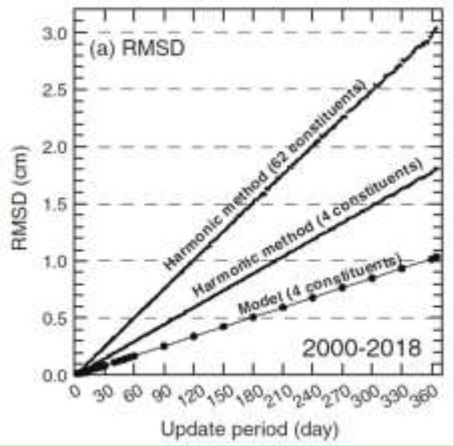
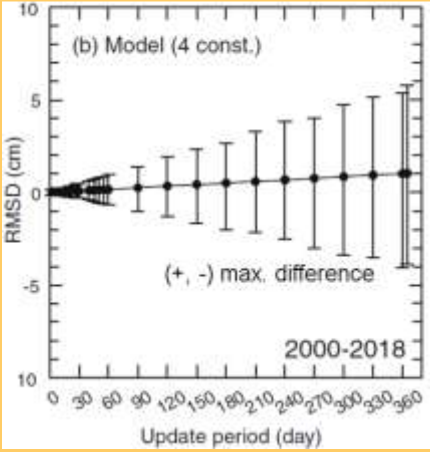
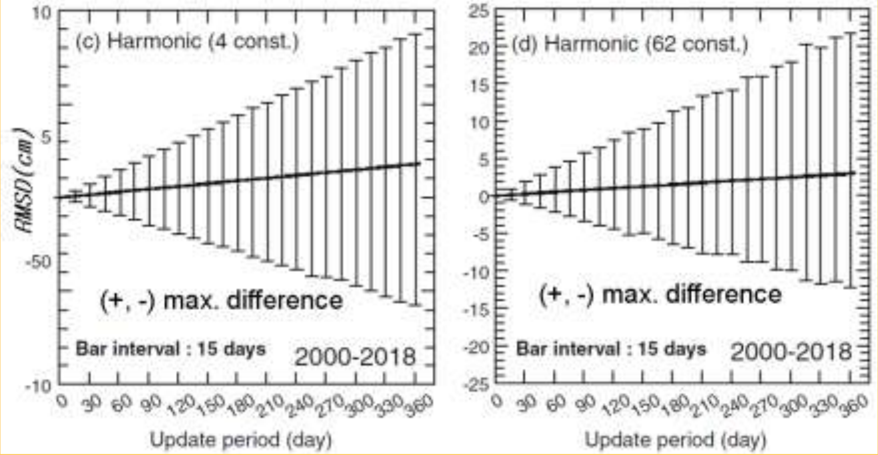
Experiments for NMC updating



Mokpo (Model)



Incheon (HPM)



The small errors created by using correct V_o values without NMC updates cannot be ignored in storm-surge forecast modeling for tidally-dominated coastal environments.

Conclusions

- **Comparisons between the estimation using different parameterizations for five astronomical variables show that these methods yield essentially the same results for the period 1900-2099, revealing all are applicable for tidal forecasting simulation.**
- **NMC for a prediction model needs updating within <30 days for accurate perpetual interannual tidal predictions and tide-storm-surge predictions requiring centimeter accuracy, for tidally-dominated coastal regimes.**


“ Thank you very much
for your attention”